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Heat transfer correlation for natural convection in a meniscus-shaped cavity and its application to contact melting process

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Abstract—Heat transfer correlation for the steady-state natural convection in a meniscus-shaped cavity was established. It was aimed at providing more accurate data for the contact melting processes in both cylindrical and spherical geometries. A closed-form exact conduction solution was also presented for the cylindrical case. It was found that normalized circumferential distribution of Nusselt number was approximately $Nu/Nu_0 \sim \cos\phi$, which confirmed successfully the shape-preserving character of the top solid surface. A sample application revealed that the convection melting in a cylindrical enclosure was estimated somewhat higher than that analyzed in the previous study. Admitting that natural convection is of secondary importance during contact melting, the present heat transfer correlation, combined with the elegant lubrication theory available elsewhere, would provide a more accurate prediction without solving complicated governing equations case by case. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

When a cylindrical enclosure or spherical capsule containing a phase-change medium is used as a thermal storage unit, the inward melting normally proceeds in such a way that the unmelted solid core is continuously moving downward within its own melt on account of gravity, thereby creating a thin liquid film layer between the core and the enclosing wall. Meanwhile, an ever-widening liquid pool develops above the solid core as melting continues. Such a contact melting process basically involves two distinct heat transfer modes; conduction in the film region and natural convection in the liquid pool. Based on that, the conduction across the film layer is of primary importance. Bareiss and Beer [1] made an excellent analysis of contact melting in a cylindrical enclosure applying the lubrication theory. Similar studies are available for spherical geometries as well [2–4]. The role of natural convection in the liquid pool has been considered also, either by an empirical approximation [1] or by solving the Boussinesq-approximated Navier–Stokes equations [3, 5]. These analyses are based on the experimental observation that the top solid surface remains at its original circular (or spherical) shape over the entire melting process, i.e. the shape-preserving character of the top solid surface has been observed.

In this study, steady-state natural convection solutions in a meniscus-shaped cavity (enclosed by two surfaces of same radius R) were obtained for both cylindrical and spherical geometries. Also, a closed-form conduction solution in the meniscus cavity was presented especially for the cylindrical case, which is useful as a reference quantity relative to convection solutions. Earlier studies [1, 3] have revealed that the liquid flow during the melting process could be considered as a series of steady-state natural convection. Based on these observations, heat transfer correlations for meniscus-shaped cavities were presented here for both geometries (such correlations may be regarded as an analogy to those for concentric cylindrical or spherical cavities). Finally as an illustration, the correlation for cylindrical case was combined with the previous analysis [1] to further examine the relative importance of convective top melting to the overall melting.

PROBLEM DESCRIPTION

A schematic of the problem considered is depicted in Fig. 1. A meniscus-shaped cavity is enclosed by two surfaces of same radius of curvature, R. The outside wall is heated isothermally at a temperature T_w, and the top solid surface is at a fixed temperature of T_f. In the present work, steady-state natural convection occurring in this meniscus-shaped cavity is analyzed to give an approximate, but acceptable, heat transfer correlation.

The conventional steady-state Boussinesq-approximated governing equations for melting problems are employed, where the dimensionless parameters are

$$Ra = \frac{g\beta(T_w - T_f)R^3}{\nu\alpha}, \quad s_0^* = \frac{s_0}{2R} \qquad (1)$$

and the phase change material under consideration is

NOMENCLATURE

Nu	Nusselt number, $q''R/k(T_w - T_f)$	ϕ	polar angle (Fig. 1)
Nu_0	Nusselt number at $\phi = 0$ (Fig. 1)	ξ, θ	tansformed coordinates for conduction analysis.
Pr	Prandtl number		
R	radius		
Ra	Rayleigh number	Superscripts	
s_0	apparent melting distances	*	dimensionless quantity (divided by $2R$)
T_f	fusion temperature		
T_w	wall temperature.	′	spherical geometry.
Greek symbols		Subscripts	
$\delta(\phi)$	melting distance measured radially	C	convection dominant
ε	cut-line thickness	K	conduction dominant.

n-octadecane (here, $Pr = 50$). It is assumed that the natural convection in the liquid pool during the contact melting can be approximated with a series of steady-state solutions corresponding to each s_0^*.

For computation, no-slip and isothermal conditions are imposed, both on the enclosing wall and on the top solid surface. Along the line of symmetry, adiabatic and full-slip conditions are used. Since a singularity can exist if the two isothermal surfaces are in contact with each other, a small cut-line is arbitrarily introduced, as illustrated in Fig. 1. For convenience, no-slip and adiabatic conditions are applied at this cut-line, which implies the occurrence of a linear temperature profile as well as the insignificance of flow effect near the cut-line. This approach was justifiable because the effect of varying ε (i.e. the length of cut-line) from $\varepsilon = 0.01R$ to $0.03R$ is found to be noticeable only in the vicinity of the cut-line. For both cylindrical and spherical geometries, heat transfer correlations are obtained by analyzing numerical results for $Ra = 10^5$, 10^6, 10^7 and various s_0^*. A detailed computational procedure can be found elsewhere [6].

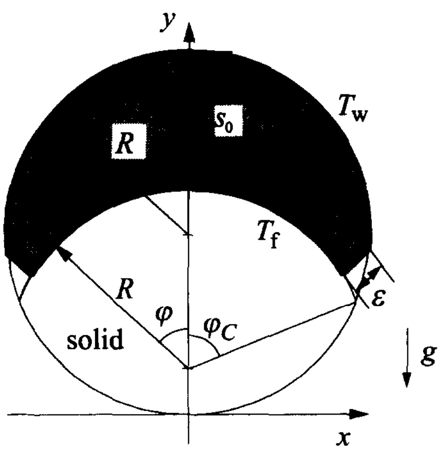

Fig. 1. Schematic representation of the meniscus-shaped enclosure.

RESULTS AND DISCUSSION

Closed-form solution for conduction case

For the cylindrical geometry, an analytical solution can be obtained using the following conformal mapping [7]:

$$\zeta + i\theta = \ln\left(\frac{x + x_c - i(y - y_c)}{x - x_c + i(y - y_c)}\right), \tag{2}$$

where $\zeta = \ln(r_2/r_1)$ and θ designates the subtended angle, i.e. $\theta = \angle\mathrm{CPD}$ (see Fig. 2). The two surfaces of the cylinder wall and solid-top are then mapped onto two horizontal lines in the (ζ, θ) plane, as shown in Fig. 2. Note that the conduction solution in the (ζ, θ) plane is simply a linear profile. This means that a set of points with the same subtended angle lie on an isotherm in the physical plane. Therefore, the steady-state conduction solution can be written as

$$\frac{T - T_f}{T_w - T_f} = \frac{\theta_2 - \theta}{\theta_2 - \theta_1}, \tag{3}$$

where, from the geometrical relation, $\theta_1 = \cos^{-1} s_0^*$ and $\theta_2 = \pi - \theta_1$. The local Nusselt number of the top solid surface at $\phi = 0$ then can be found by differentiating equation (3), and rearranging yields

$$Nu_K \equiv (Nu_0)_{\mathrm{cond}} = \frac{1}{\theta_2 - \theta_1}\cot(\theta_1/2). \tag{4}$$

This expression will be used in quantifying the significance of natural convection relative to conduction and in establishing a general heat transfer correlation.

Cylindrical case

An earlier elegant analysis of Bareiss and Beer [1] mainly considered conduction melting over the thin film layer, while natural convection was treated with an empirical relation, $Nu_0 = 0.2Ra^{1/4}$. However since the influence of the natural convection on the heat transfer is a strong function of gap width s_0^*, it is necessary to build up a proper heat transfer cor-

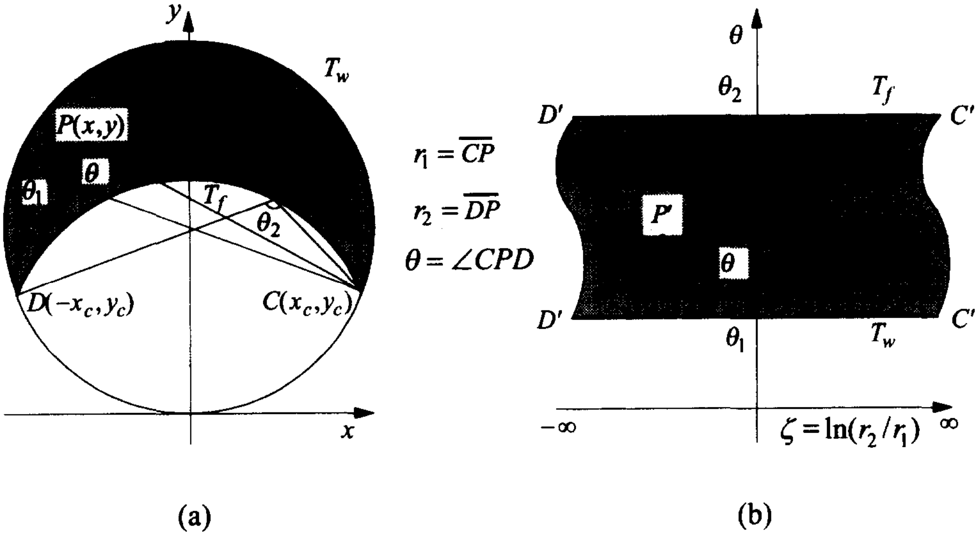

Fig. 2. Transformation used for exact conduction solution: (a) physical domain, (b) transformed domain.

relation to more accurately handle the influence of natural convection. By analyzing our numerical results and using a canonical form similar to that in Gobin and Benard [8], the following correlation was established:

$$Nu_0 = Nu_K + (Nu_C - Nu_K) \times \left[1+\left(\frac{1}{0.22Ra^{1.04}\cdot s_0^{*4.11}}\right)^2\right]^{-1/2} \quad (5)$$

where Nu_K is given by equation (4) and Nu_C is curve-fitted into

$$Nu_C = 0.33Ra^{1/4}. \quad (6)$$

In the above, Nu_0 represents the local Nusselt number of the top solid surface at $\phi = 0$ (see Fig. 1).

Figure 3 displays how well the heat transfer correlation fits with the numerical results for various s_0^*. It is interesting to note that there exists an extremum near $s_0^* \cong 0.1$ at high Ra. This arises from a compromise between two conflicting facts; as s_0^* increases, natural convection tends to intensify, whereas the thermal stratification is more pronounced. In all cases, Nu_0 is nearly close to an asymptote of Nu_C when $s_0^* \geqslant 0.2$. Since the magnitude of Nu_0 corresponds to the rate of convective top melting, the presence of Nu_0-asymptote substantiates the experimental observation that the melting distance at the top varies almost linearly with time [1]. However, it is noted that our Nu_C differs by a constant from that employed in Bareiss and Beer [1]. Prasad and Sengupta [5] also gave a similar remark on this point, but made no attempt to present heat transfer correlations.

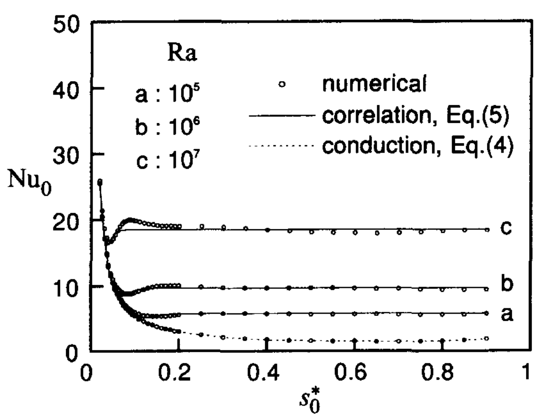

Fig. 3. Numerical results for the local Nusselt number and the corresponding correlation for the cylindrical geometry.

Next, by denoting $\delta(\phi)$ the radial melting distance of the top solid surface, it can be inferred from the geometrical constraint that shape-preserving will occur if

$$\frac{d\delta}{dt}(\phi) = \frac{d\delta}{dt}(0)\cos\phi \quad (7)$$

for both cylindrical and spherical cases. Since Nu is proportional to $d\delta/dt$, this can be rephrased so that $Nu/Nu_0 = \cos\phi$ when the top solid surface retains its circular (or spherical) shape. Our numerical results for the circumferential distribution of Nu are therefore plotted in Fig. 4 after normalized by Nu_0. Consistent with the foregoing discussion, the collection curves of Nu/Nu_0 for various s_0^* nearly coincide with the $\cos\phi$ curve with $\phi < 10^0$. It is noted that the maximum deviation was below 10% for a range of $\phi < \phi_C/2$ (i.e. half of the top solid surface). The results in Fig. 4 therefore explain, within the first approximation, the cause of shape-preserving behavior observed experimentally [1, 9].

Spherical case

In a manner similar to the cylindrical case, the heat transfer correlation for natural convection is developed as

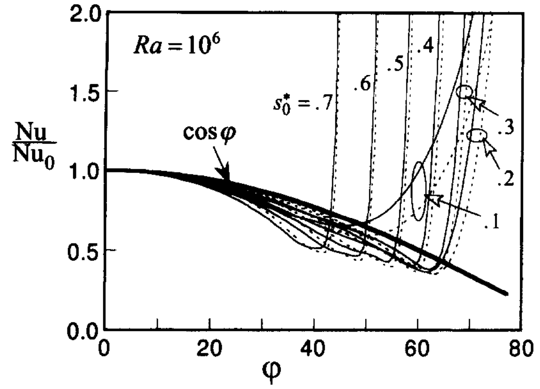

Fig. 4. Normalized circumferential distribution of the local Nusselt number at the top solid surface to explain the shape-preserving behavior; solid lines for cylindrical case and dotted lines for spherical case.

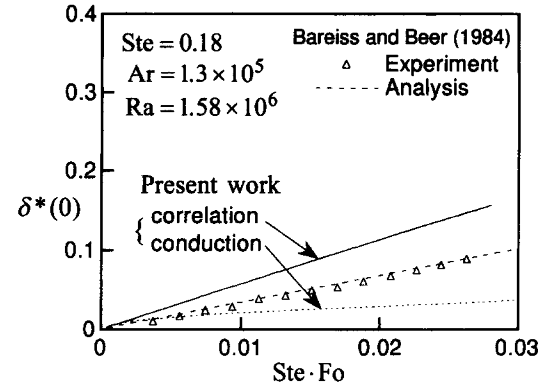

Fig. 5. The melting distance δ^* (0) due to natural convection from the present study and Bareiss and Beer [1], where Ste, Ar and Fo represent the Stefan, Archimedes and Fourier numbers, respectively, defined as in ref. [1].

$$Nu'_0 = Nu'_K + (Nu'_C - Nu'_K) \times \left[1 + \left(\frac{1}{0.059 Ra^{0.655} \cdot s_0^{*2.56}}\right)^2\right]^{-1/2}. \quad (8)$$

However an exact conduction solution was not attempted for this case; instead, an analogy was derived from the numerical results such that

$$\frac{Nu'_K}{Nu_K} \approx 1 + \sin s_0^* \quad (9)$$

and

$$Nu'_C = 0.51 Ra^{1/4}. \quad (10)$$

For this spherical case, agreement between the numerical results and the correlation is found to be equivalent to that shown in Fig. 3. It is noted that, since the rate of decrease in the unmelted volume to that of the contact area is much higher for spherical geometry, both Nu'_C and Nu'_K are at larger values. This is indeed consistent with the fact that a spherical geometry is advantageous over a cylindrical one, from the standpoint of energy storage efficiency.

A sample application

As an illustrative example, the heat transfer correlation is then incorporated into the analysis of Bareiss and Beer [1] to evaluate and compare the portion of top melting due to natural convection. Figure 5 shows the results from our new correlation and those of Bareiss and Beer [1]. It is evident that the predictions from the two analyses are all above that based on conduction only. While the experimental data of Bareiss and Beer [1] agree favourably with their prediction, our new correlation overestimates the convective top melting, being about 15% of the total melting. This is due mainly to the difference in estimating Nu_0 in two studies. A similar analytical investigation with spherical geometry [3] also has a trend of overestimation when compared with the experimental data [9].

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